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Upper Bound on the Lightest Higgs Mass in Supersymmetric Theories

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Abstract

The problem of the lightest Higgs boson mass in the next-to-minimal supersymmetric standard model (NMSSM) is investigated. Assuming the validity of the perturbation theory up to unification scales and using the recent experimental results for the top quark mass, the restrictions on the NMSSM coupling constants are obtained. These restrictions are used to make the predictions for the lightest Higgs mass, which are compared to those of the minimal supersymmetric standard model (MSSM).

1. The aim of this letter is to investigate the lightest Higgs boson mass problem in the next-to-minimal supersymmetric standard model (NMSSM). This model contains an additional Higgs singlet, as compared to the minimal supersymmetric standard model (MSSM). The cause of consideration of the NMSSM is connected in particular with desire to avoid an explicit mass term for the Higgs doublets in a superpotential [1]. There is also an interesting possibility to realize the scenario with spontaneous CP-violation in this model [2]-[6]. It is known that in the minimal supersymmetric standard model such a scenario [7] contradicts with existing experimental constraints on the MSSM neutral Higgs masses [8, 9], while in the NMSSM this contradiction can be avoided in different ways [5, 6]. The experimental constraints on Higgs masses play important role also for the case of the absence of spontaneous CP violation. The recent LEP experiments give the lower bound on the lightest MSSM Higgs mass $m_h > 75$GeV [8]. Depending on the Higgs vacuum expectation values (vev's) ratio $\tan\beta$, this bound can be much stronger. Thus, one derives [9] $m_h > 88$GeV for $\tan\beta \leq 2$. This makes the MSSM close to being excluded, when the low $\tan\beta$ scenario is considered. It is interesting to analyze how the situation is looked in the NMSSM.

2. The next-to-minimal supersymmetric standard model had been widely studied in literature [10]-[20]. It was noted already in ref. [10] that the upper bound on the lightest NMSSM Higgs particle mass can be different from the one in the MSSM. On the other hand, the NMSSM Higgs sector is described (even at tree level) by large enough number of unknown parameters, which makes the analysis of Higgs masses very difficult. Today only the case, when the supersymmetry breaking parameters of the theory are related by universality conditions, has been investigated in details [14, 17, 18]. Requiring for the physical minimum of the scalar potential to be the global one and taking into account the experimental bounds on the NMSSM particles masses, it was found that the Higgs singlet sector is decoupled and - due to smallness of the relevant coupling constants - the predictions for the detectable (nonsinglet) Higgs bosons are almost the same as those of the MSSM. However, as it was recently pointed out [19], the NMSSM parameters space, where the scenario with the universal supersymmetry breaking can be realized, is strongly restricted, if it is required for the physical minimum of the scalar potential to be the global one. One can avoid such a strong constraints on parameters of theory either assuming that the physical minimum of the potential is a local one, or relaxing the universality conditions at high energy scales.
In this letter no universality conditions on the supersymmetry breaking parameters are assumed. In this case either only some special regions of parameters space of theory are investigated or an upper bound on the mass of the lightest Higgs boson, as a function of some parameter(s) of theory, is only found. Here the latter strategy is used: an upper bound on the lightest Higgs mass, as a function of $\tan\beta$, is obtained. The only restrictions on parameters of theory, which are taken into account, are constraints on the coupling constants, coming from the assumption of the absence of new physics between the supersymmetry breaking scale (M_{SUSY}) and unification scales (M_G), as well as from the recent experimental results for the top quark mass. The effects, connected with new

experimental results for the top quark mass, play important role, when being taken into account appropriately. These effects together with two-loop order corrections to the lightest Higgs mass lead to the predictions for this particle mass, which are different by about (20-25)GeV from those, reported in literature previously. The derived upper bound on the lightest Higgs mass is compared to the one of the MSSM.

3. The NMSSM superpotential is obtained from the MSSM one by making in the latter one the following replacement:

$$\mu \bar{H}_1 \bar{H}_2 \rightarrow \lambda \bar{N} \bar{H}_1 \bar{H}_2 + \frac{\kappa}{3} \bar{N}^3 \tag{1}$$

(here $\bar{H}_1$ and $\bar{H}_2$ are the Higgs doublets and $\bar{N}$ is the Higgs singlet superfields with scalar components H_1, H_2, N). Thus, one avoids an explicit mass term in the superpotential - such a term was needed, to provide for spontaneous breaking of the $SU(2)_L \times U(1)_Y$ symmetry. Instead one introduces the additional complex degree of freedom, connected with the $SU(2)_L \times U(1)_Y$ Higgs singlet N, as well as two Yukawa-type coupling constants, λ and κ. These couplings enter the Higgs bosons mass matrices in nontrivial way and to find the restrictions on the lightest Higgs mass, one has to obtain some constraints on λ and κ and/or correlations between them. Such constraints can be obtained from the NMSSM renormalization group equations (RGE's) analysis. Such an analysis in the standard model and its extensions has been performed in ref's [21]-[30] - there various approaches had been developed.
In this letter it is assumed that all the (gauge and Yukawa-type) couplings are small between the electroweak breaking scale $\sim$ 100GeV (this scale is identified also with Z boson mass M_Z or with the top quark mass m_t) and unification scales ($M_G \sim (10^{16} - 10^{18})$GeV), so that the perturbation theory is applied. This condition leads to restrictions on coupling constants of theory. As a result, one obtains some bounds on the lightest Higgs boson mass.
During the renormalization group equations analysis I neglect the supersymmetric particles mass thresholds effect, assuming that supersymmetric RGE's are valid from the electroweak breaking scale up to unification scales. I perform the analysis at one-loop level - as the numerical analysis shows, the two-loop corrections do not exceed few percents, i.e. are of the same order, as the inaccuracy of approach, connected with the uncertainty of choice of the cut-off scale (whether it is chosen $\sim 10^{16}$GeV or, say, $\sim 10^{18}$GeV).

4. Let us discuss briefly the problem of the lightest Higgs boson mass in the MSSM and its simplest extension with the additional Higgs singlet. In the minimal supersymmetric standard model [31, 32] there are five physical Higgs states: two CP-even, one CP-odd and one complex charged Higgs states. At tree level these particles masses are described by only two unknown parameters: the CP-odd Higgs mass m_A and the Higgs doublets vev's ratio $\tan\beta = v_2/v_1$. As a result, Higgs boson masses and the restrictions on them are given by compact expressions. In particular, for the lightest (CP-even) Higgs mass it is obtained that $m_h < M_Z|\cos 2\beta| < M_Z$. Notice however that this relation is

violated, when radiative corrections to the Higgs potential are taken into account. Usually one takes into account only the radiative corrections, connected with the top and bottom quarks and squarks loops - this is because of largeness of the top and (for large $\tan\beta$) bottom Yukawa couplings, as compared to other coupling constants. When being taken into account at one-loop level [33]-[37], these corrections can increase the lightest Higgs mass about (30-60)GeV, depending on parameters space of theory. Recently the two-loop order corrections to the lightest Higgs mass had been also calculated [38]-[40]. It was found that these corrections are important too: they can lower m_h more than 10GeV.

The above-mentioned radiative corrections to the lightest Higgs mass depend on the following set of parameters: the top mass m_t, the stop masses $m_{\tilde{t}_1}$, $m_{\tilde{t}_2}$, the sum of Higgs doublets vev's squared $\eta^2 = v_1^2 + v_2^2 = (174GeV)^2$, the Higgs vev's ratio $\tan\beta$, the QCD coupling constant $\alpha_3 = g_3^2/(4\pi)$, as well as (via the stops mixing effects) on the stop trilinear supersymmetry breaking parameter A_t and the explicit mass parameter μ. In this paper it is assumed that $m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2 \ll m_{\tilde{t}_1}^2 + m_{\tilde{t}_2}^2$ (then stops masses are identified usually with the supersymmetry breaking scale). In this approximation one obtains at two-loop level [39, 40]

$$\begin{aligned} m_h^2 < M_Z^2 \cos^2 2\beta \left(1 - \frac{3}{8\pi^2}\frac{m_t^2}{\eta^2}\log\frac{M_{SUSY}^2}{m_t^2}\right) + \frac{3}{4\pi^2}\frac{m_t^4}{\eta^2}\left[\frac{1}{2}X_t + \right. \\ \left. + \log\frac{M_{SUSY}^2}{m_t^2} + \frac{1}{16\pi^2}\left(\frac{3}{2}\frac{m_t^2}{\eta^2} - 32\pi\alpha_3\right)\left(X_t \log\frac{M_{SUSY}^2}{m_t^2} + \log^2\frac{M_{SUSY}^2}{m_t^2}\right)\right] \end{aligned} \tag{2}$$

where

$$X_t = \frac{2(A_t - \mu\cot\beta)^2}{M_{SUSY}^2}\left(1 - \frac{(A_t - \mu\cot\beta)^2}{12M_{SUSY}^2}\right) \tag{3}$$

It is easy to see that $X_t \leq 6$ - the maximum value of X_t is obtained, when $A_t - \mu\cot\beta = \sqrt{6}M_{SUSY}$. In this paper two values of X_t are considered: $X_t = 0$ - the case of so-called no-mixing, and $X_t = 6$ - the case of so-called maximal mixing.

In the next-to-minimal supersymmetric standard model there are additional one CP-even and one CP-odd Higgs degrees of freedom, as compared to the MSSM. At tree level the 3×3 symmetric mass matrix for CP-even Higgs fields Φ_1, Φ_2 and N_1 [10]-[13], [20] is given by formula (A1) of Appendix. Usually one takes $\Phi_1 = \frac{1}{\sqrt{2}}(Re(H_1^0) - v_1)$, $\Phi_2 = \frac{1}{\sqrt{2}}(Re(H_2^0) - v_2)$ and $N_1 = \frac{1}{\sqrt{2}}(Re(N) - v_3)$, where v_3 is the singlet vev. In this paper a little bit different representation is used, namely,

$$\begin{aligned} \Phi_1 &= \frac{1}{\sqrt{2}}(Re(H_1^0)\cos\beta + Re(H_2^0)\sin\beta - \eta) \\ \Phi_2 &= \frac{1}{\sqrt{2}}(-Re(H_1^0)\sin\beta + Re(H_2^0)\cos\beta) \\ N_1 &= \frac{1}{\sqrt{2}}(Re(N) - v_3) \end{aligned} \tag{4}$$

Such a choice of the representation of CP-even Higgs fields makes the matrix (A1) more transparent for the qualitative analysis.

At tree level the upper bound on the lightest (CP-even) Higgs boson mass is the following:

$$m_h^2 < M_Z^2 \cos^2 2\beta + \lambda^2 \eta^2 \sin^2 2\beta \tag{5}$$

where the right hand side of eq. (5) is the maximum value of the lowest eigenvalue of 2×2 upper block ($M^2_{S_{ij}}, i, j = 1, 2$) of the matrix (A1) [11, 13]. In other words, the upper bound (5) is saturated, when the singlet sector is decoupled[1] and the detectable Higgs bosons masses are described by the 2×2 MSSM upper block of the matrix (A1). Like in the MSSM, there is only one (nonsinglet) Higgs boson with the mass of the order of electroweak breaking scale, when the upper bound on m_h is saturated[2].

Even in the region of parameters space, where the singlet sector is decoupled, there is a difference between the radiative corrections to the lightest Higgs mass in the MSSM and those in the NMSSM. Such a difference arises due to the loops, connected with the term $(\lambda^2 - g_2^2/2)|H_1 H_2|^2$ of the Higgs potential [11]. The contribution of these loops is proportional to λ^4, $\lambda^2 g_2^2$, $\lambda^2 g_1^2$, $\lambda^2 h_t^2$, etc. (g_2 and g_1 are respectively $SU(2)_L$ and $U(1)_Y$ gauge coupling constants, h_t is the top quark Yukawa coupling).

In the next section it is obtained that λ is of the order of weak coupling constant g_2 or smaller. This allows one to neglect all aforementioned corrections, except of those, proportional to $\lambda^2 h_t^2$. These latter corrections arise due to the top quark and squark loops. The contribution of these loops is generalized in the case of the NMSSM in the following way.

As it was mentioned above, both in the MSSM and in the NMSSM the upper bound on the lightest Higgs mass is saturated, when there is only one light (detectable) Higgs boson with the mass of the order of electroweak breaking scale. Then at tree level one may write $m_h^2 < m^2_{h_{max}} = 4\lambda_{SM}\eta^2$, where[3] $\lambda_{SM} = M_Z^2 \cos^2 2\beta/(4\eta^2)$ and $\lambda_{SM} = M_Z^2 \cos^2 2\beta/(4\eta^2) + \lambda^2 \sin^2 2\beta/4$ for the MSSM and the NMSSM respectively. As for the radiative corrections, they can be subdivided in following three parts:

a) the corrections, coming from the one-loop renormalization of λ_{SM},
b) one-loop corrections, connected with the stops mixing effect,
c) next-to-leading order (two-loop order) corrections.

The corrections a) can be read off from the standard model RGE's [29]: one obtains that

$$\lambda_{SM} \to \lambda_{SM} \left(1 - \frac{3}{8\pi^2} h^2_{t_{SM}} \log \frac{M^2_{SUSY}}{m_t^2} \right) + \frac{3}{16\pi^2} h^4_{t_{SM}} \log \frac{M^2_{SUSY}}{m_t^2}$$

[1]The necessary condition for this to occur is $M^2_{S_{13}} \to 0$. If even $M^2_{S_{22}}, M^2_{S_{33}} \gg M^2_{S_{11}} \sim M_Z^2$, one derives $m_h^2 = M^2_{S_{11}} - (M^2_{S_{13}})^2/M^2_{S_{33}}$, so that the upper bound (5) is saturated, when $M^2_{S_{13}} \to 0$. This occurs if either $\lambda \to 0$, or $v_3 \to 0$ (this case is disfavoured by the experimental bounds on the lightest chargino mass) or in the case of fine-tuning of the relevant NMSSM mass parameters. In other words, the upper bound (5) is saturated only in narrow region of the NMSSM parameters space. It is worth to stress also that this is the upper bound on the lightest *detectable* (nonsinglet) Higgs boson.

[2]Generally speaking, this rule is violated for $\tan\beta \gg 1$: then the upper bound on m_h can be saturated also when more than one Higgs bosons with the masses $\sim M_Z$ exist. However these values of $\tan\beta$ are out of the interest: as it is easy to understand, the difference between the models predictions occurs due to the coupling λ, which enters the above-mentioned 2×2 block with the factor $\sin 2\beta$.

[3]The choice of the coefficient in front of λ_{SM} depends on the choice of the corresponding coefficient of the standard model Higgs potential. Here the same notation for λ_{SM}, as in ref. [29], is used.

where $h_{t_{SM}} = m_t/\eta$. The corrections b) and c) depend only on the top and stop sectors of Lagrangians of the models. These sectors (and hence the corrections b) and c)) are almost the same in both of models. The difference is only that the mass parameter μ is replaced by the product λN (or vice versa). Respectively, in equation (3) one has to make the replacement $\mu \to \lambda v_3$.
Using the above-mentioned analogy between the models, one obtains the following bound on the lightest NMSSM Higgs particle:

$$m_h^2 < (M_Z^2 \cos^2 2\beta + \lambda^2 \eta^2 \sin^2 2\beta) \left(1 - \frac{3}{8\pi^2} \frac{m_t^2}{\eta^2} \log \frac{M_{SUSY}^2}{m_t^2}\right) + \frac{3}{4\pi^2} \frac{m_t^4}{\eta^2} \left[\frac{1}{2} X_t + \right.$$
$$\left. + \log \frac{M_{SUSY}^2}{m_t^2} + \frac{1}{16\pi^2} \left(\frac{3}{2} \frac{m_t^2}{\eta^2} - 32\pi\alpha_3\right) \left(X_t \log \frac{M_{SUSY}^2}{m_t^2} + \log^2 \frac{M_{SUSY}^2}{m_t^2}\right)\right] \qquad (6)$$

One can see from comparison of (6) to (2) that the difference between the models predictions can occur due to the term, proportional to $\lambda^2 \eta^2 \sin^2 2\beta$. This term is important for low values of $\tan\beta$. Therefore I restrict myself by consideration of $\tan\beta \le 6$. To estimate the difference between the models predictions, one must find some constraints on the coupling constant λ. As it was mentioned above, such constraints are derived from the renormalization group equations analysis.
One assumes often (see e.g. [20]) that $|\lambda| < 0.87$- the infrared (IR) quasi-fixed point, derived in the limit $h_t = 0$. Such an assumption is not quite correct: the restrictions on λ become stronger, when $h_t \sim 1$ [11, 13]. As it is shown in the next section, the restrictions on λ for the experimentally allowed range of the top mass are about 20% different from the bound, reported above.

5. For the low $\tan\beta$ scenario the RGE's for $SU(3)_c$, $SU(2)_L$, $U(1)_Y$ couplings, g_3, g_2 and g_1 respectively, and Yukawa-type couplings h_t, λ and κ are the following [30, 41]:

$$\begin{aligned} 16\pi^2 \frac{d\kappa}{dt} &= 6\kappa(\kappa^2 + \lambda^2) \\ 16\pi^2 \frac{d\lambda}{dt} &= \lambda(2\kappa^2 + 4\lambda^2 + 3h_t^2 - \frac{3}{5} g_1^2 - 3g_2^2) \\ 16\pi^2 \frac{dh_t}{dt} &= h_t(6h_t^2 + \lambda^2 - \frac{13}{15} g_1^2 - 3g_2^2 - \frac{16}{3} g_3^2) \\ 16\pi^2 \frac{dg_i}{dt} &= -c_i g_i^3 \end{aligned} \qquad (7)$$

where $c_1 = -33/5$, $c_2 = -1$, $c_3 = 3$ and $t = 1/2\ \log(Q^2/M_Z^2)$ (as it was mentioned above, I neglect the supersymmetric particles mass thresholds effect, considering these RGE's valid from the electroweak breaking scale up to unification scales). The behaviour of gauge couplings is determined by their experimental values at the electroweak breaking scale [42]: $g_1(m_t) \approx 0.46$, $g_2(m_t) \approx 0.65$, $g_3(m_t) \approx 1.22$. Some restrictions on the top quark Yukawa coupling is found from the experimental constraints on the top mass. According the recent experimental data [42], $m_t^{pole} = (173.8 \pm 5.2)$GeV. One has to transform the

pole top mass into the on-shell top mass

$$m_t \equiv m_t(m_t) = h_t(m_t)\eta \sin\beta \tag{8}$$

to make the predictions for h_t. This transformation is done, using the well-known relation between the pole and the on-shell top masses [43];

$$m_t(m_t) = \frac{m_t^{pole}}{1 + \frac{g_3^2}{3\pi^2}} = (165 \pm 5)GeV \tag{9}$$

where in eq. (9) only the leading order QCD gluon corrections are taken into account. Higher order QCD corrections as well as the stop/gluino loops corrections [44, 45] modify the on-shell top mass by about 2GeV - they can also cancel each other. Here I neglect these corrections: to take into account the stop/gluino loops effects, one has to determine both the stops masses and the stops mixing angle. This deviates from the framework of this letter.

As it follows from equations (8), (9), $0.92/\sin\beta \le h_t(m_t) \le max(h_{t_{max}}, 0.98/\sin\beta)$, where $h_{t_{max}}$ is the triviality bound on $h_t(m_t)$. This bound, as well as bounds on λ and κ (λ_{max} and κ_{max}) are found from the analysis of RGE's for these coupling constants.

The procedure is the following. One evolutes (numerically) λ, κ, h_t and g_1, g_2, g_3 according their renormalization group equations from the electroweak breaking scale up to the GUT scale $\sim 10^{16}$GeV. It is required at whole considered energy range for the couplings to be small enough to perturbation theory being applied. More precisely, the following condition on λ, κ and h_t must be satisfied:

$$\lambda^2(Q^2) < 4\pi,\ \kappa^2(Q^2) < 4\pi,\ h_t^2(Q^2) < 4\pi \tag{10}$$

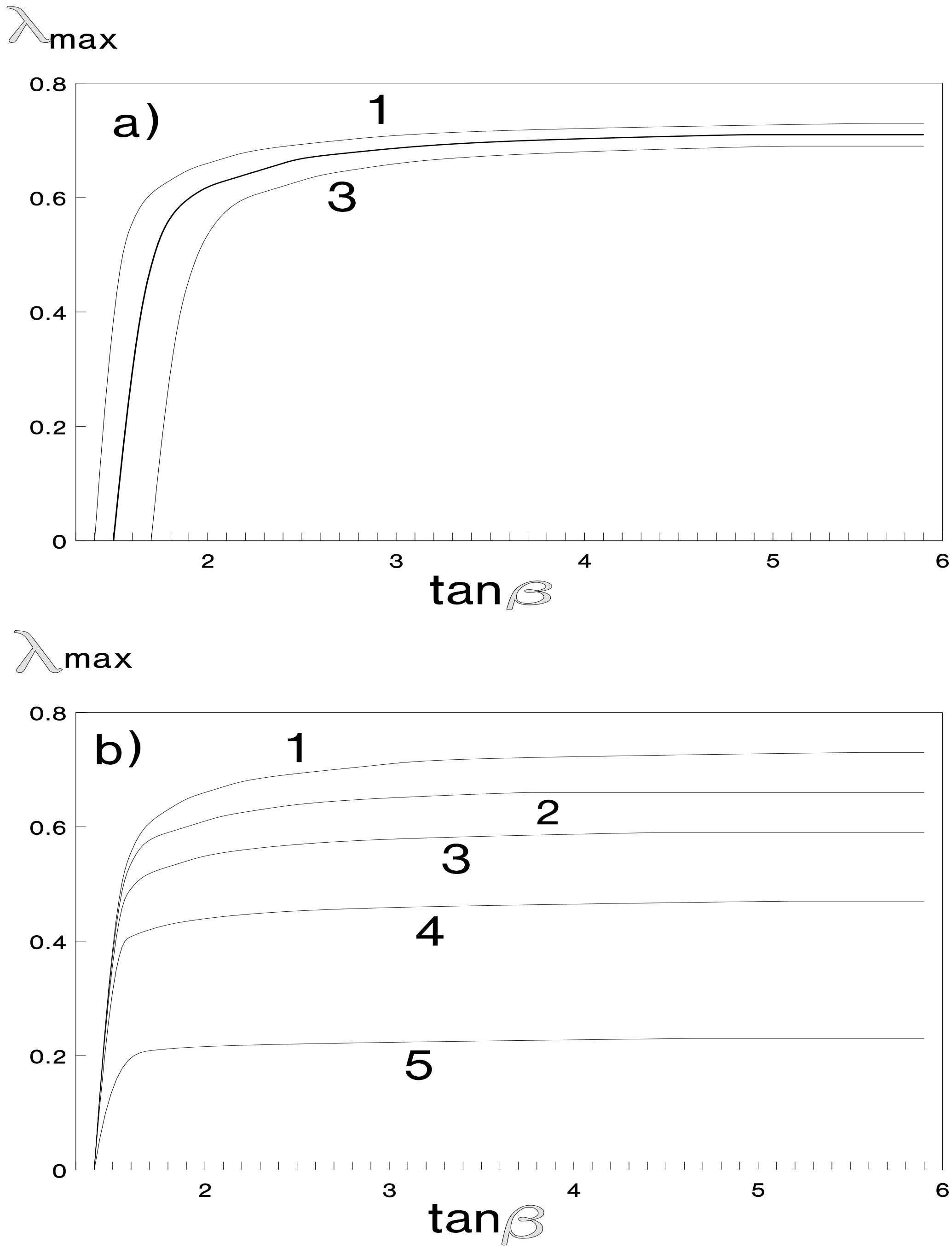
The conditions (10) put some constraints on coupling constants at the electroweak breaking scale. Thus, one obtains $h_t(m_t) < 1.15$ and $|\kappa(m_t)| < 0.65$. The reported upper bound on $h_t(m_t)$ is valid also for the MSSM: this bound is obtained in the limit $\lambda = 0$, where the RGE for h_t is reduced to the MSSM one. Using equation (8), one can also transform (for fixed value of the top mass) the restriction on $h_t(m_t)$ to the lower bound on $\tan\beta$. Clearly, this lower bound is the same both for the MSSM and for the NMSSM.

The restrictions on λ, as functions of $\tan\beta$, are presented in Fig, 1. As one can see from this figure, $|\lambda(m_t)| \le 0.7$ - this bound is obtained for $\tan\beta \ge 3$ and $\kappa(m_t) = 0$. Thus, the absolute upper boundary on λ, derived here, is about 20% lower than IR-quasi fixed point $\lambda = 0.87$. Such a difference arises, when the constraints on $h_t(m_t)$, coming from the recent experimental results for the top quark mass, are taken into account.

The behaviour of λ_{max} with $\tan\beta$ is nontrivial. As one can see from Fig, 1, upper bound on λ remains almost unchanged, when $3 \le \tan\beta \le 6$, it decreases slowly, when $2 \le \tan\beta \le 3$ and, finally, λ_{max} is dropped to zero for the lowest allowed values[4] of $\tan\beta$.

[4]In other words, λ_{max} is highly sensitive to $h_t(m_t)$, when the latter is close to its upper boundary. On the other hand, the RGE's analysis approach is not able to make predictions for the couplings with such an accuracy. This makes the predictions for λ unreliable, when the lowest values of $\tan\beta$ are considered. Notice however that our main results are obtained for $\tan\beta \approx 2$, where such a problem does not exist.

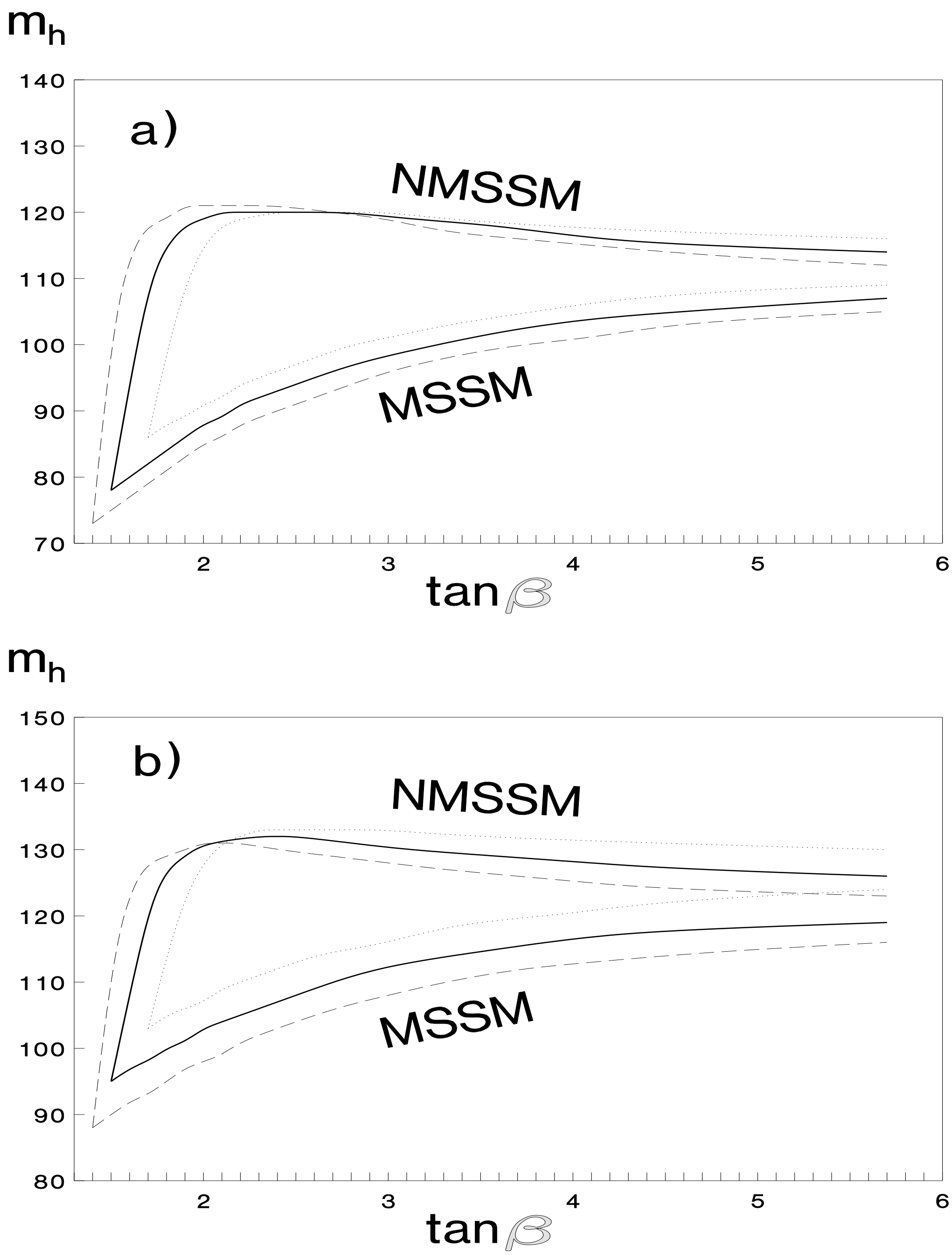
Figure 1: Upper bound on $|\lambda(m_t)|$ (λ_{max}) as a function of $\tan\beta$ a) for $\kappa(m_t) = 0$ and $m_t = 160$GeV (line 1), $m_t = 165$GeV (solid line) and $m_t = 170$GeV (line 3), b) for $m_t = 160$GeV and $|\kappa(m_t)| = 0; 0.3; 0.4; 0.5; 0.6$ (lines 1,2,3,4,5 respectively)



It is interesting to investigate also, how the restrictions on λ are affected by the experimental error in the top mass. For this purpose I find λ_{max} for $m_t = 160$GeV, $m_t = 165$GeV and $m_t = 170$GeV. As one can see from Fig, 1a), either $\lambda_{max}(\tan\beta)$ or $\tan\beta(\lambda_{max})$ vary only weakly, when the top mass varies form 160GeV to 170GeV. Notice however that this weak variation of λ_{max} (or $\tan\beta$) becomes nonnegligible, when being combined with the sensitivity of the lightest Higgs mass to the top mass via the radiative corrections [45, 46]. It is clear that the difference between the pole and the on-shell top masses is nonnegligible too. This is in spite of this difference affecting only weakly on λ_{max}: one obtains $|\lambda(m_t)| \leq 0.65$, when taking $m_t = 174$GeV (or $h_t(m_t) \geq 1/\sin\beta$).
The restrictions on λ, discussed above, were derived in the limit $\kappa = 0$. Notice however that they may be considered approximately valid for $|\kappa(m_t)| < 0.3$. Indeed, as one can see from Fig, 1b), the difference between the results, derived for $\kappa(m_t) = 0$ and $|\kappa(m_t)| = 0.3$ is negligible for $\tan\beta < 2$ and only weak for $\tan\beta > 2$. More precisely, for $|\kappa(m_t)| = 0.3$ one derives $|\lambda(m_t)| \leq 0.65$ instead of $|\lambda(m_t)| \leq 0.7$. For fixed values of the top mass and $\tan\beta$ such a small difference does not affect the predictions for the lightest Higgs mass. This allows one to avoid the problems, connected with the correlations between the predictions for λ and κ.

6. Let us return to the Higgs bosons masses problem in the MSSM and its simplest extension with the additional Higgs singlet. As it was discussed previously, the restrictions on the lightest Higgs mass depend on the gauge coupling constants, on the Higgs vev's ratio $\tan\beta$, on the top quark mass, on the stops masses, which are identified here with the supersymmetry breaking scale, on the parameter X_t, given by eq, (3), and (in the NMSSM) on the coupling constant λ. Here I take $1.4 \leq \tan\beta \leq 6$ (the lower bound on $\tan\beta$ is derived, using the upper bound on $h_t(m_t)$), $M_{SUSY} = 1$TeV, $X_t = 0$ and $X_t = 6$. The values of the gauge couplings and the top quark mass, as well as the restrictions on λ were presented in previous section. The derived upper bound on the lightest Higgs mass ($m_{h_{max}}$) as a function of $\tan\beta$ is presented in Fig, 2.
As one can see from this figure, the MSSM and the NMSSM upper bounds on the lightest Higgs mass are different in general. This difference (Δ_h) is maximal for $\tan\beta$ being close to 2. For lower values of $\tan\beta$ the difference between the models predictions decreases fastly to zero (together with λ_{max}). For larger values of $\tan\beta$ Δ_h decreases too: as one can see from Fig, 2, the predictions of the models tend slowly to coincide for $\tan\beta \gg 1$. The results, presented in Fig, 2, are obtained for $m_t = 160$GeV, $m_t = 165$GeV and $m_t = 170$GeV, for the cases of no-mixing (Fig, 2a) and maximal mixing (Fig, 2b). The largest difference between the models predictions is derived for $m_t = 160$GeV. Thus, in the case of no-mixing one obtains $\Delta_h \approx 40$GeV. For $m_t = 165$GeV and $m_t = 170$GeV it is obtained $\Delta_h \approx 32GeV$ and $\Delta_h \approx 25$GeV respectively. The situation is similar for the case of maximal mixing. Here one obtains $\Delta_h \approx 35$GeV, 28GeV and 22GeV respectively for $m_t = 160$GeV, 165GeV and 170GeV.
Thus, the difference between the models predictions varies about 15GeV, when the top mass varies from 160GeV to 170GeV. This is caused by different behaviour of the lightest Higgs mass upper bound with the top mass in the MSSM and in the NMSSM. In the

Figure 2: Upper bound on the lightest Higgs mass in the MSSM and the NMSSM a) for $X_t = 0$ (no-mixing) b) for $X_t = 6$ (maximal mixing). The results are derived for $m_t = 160 GeV$ (dashed line), $m_t = 165$GeV (solid line) and for $m_t = 170$GeV (dotted line).



MSSM the experimental error ± 5GeV in the top mass leads to approximately the same error in $m_{h_{max}}$. On the contrary, in the NMSSM for low values of $\tan\beta$ $m_{h_{max}}$ either remains unchanged or decreases with the increasing of the top mass.
There is also the difference in the behaviour of the models predictions with $\tan\beta$. As one can see from Fig, 2, in the MSSM $m_{h_{max}}$ increases monotonously with $\tan\beta$, so that the absolute upper bound on m_h is reached for $\tan\beta \gg 1$. On the contrary, in the NMSSM the absolute upper bound is saturated for $\tan\beta = 2 - 2.5$. Such a difference in the models predictions is connected, in particular, with the difference between the pole and the on-shell top masses, as well as with two-loop order radiative corrections to the lightest Higgs mass. On the absence of these two effects the NMSSM absolute upper bound on m_h is saturated for $\tan\beta \gg 1$ too [5].
It is interesting to compare the results, derived here in the case of the NMSSM, to those, reported in literature previously. Let us start with the case of no-mixing - in this case it is obtained that $m_h < 120$GeV. The study of this case, using the RGE's for the Higgs potential effective coupling constants, has been carried out in ref. [11] - there the result $m_h < 145$GeV had been obtained. As one can see, the difference between the results is 25GeV. This difference is caused both by stronger restrictions on λ and by next-to-leading order corrections to the lightest Higgs mass. These two effects modify the predictions for $m_{h_{max}}$ by 15GeV and 10GeV respectively.
In the case of maximal mixing it is obtained $m_h < (130 - 135)$GeV (depending on the top mass). This result is compared to those, reported in ref's [12, 13, 20] - there the upper bound $m_h < 155$GeV is obtained. Again, the difference between the results is (20-25)GeV. However, unlike the case of no-mixing, the dominant contribution to this difference (15GeV) comes now from two-loop order radiative corrections.

7. Thus, the problem of the restrictions on the lightest Higgs boson mass in the next-to-minimal supersymmetric standard model has been analyzed. To obtain these restrictions, the constraints on coupling constants of theory were taken into account. These constraints come from the renormalization group equations analysis and from the recent experimental results for the top mass. The experimental constraints on the top mass are very important: they make the restrictions on coupling constants much stronger. The another important effect is connected with two-loop order radiative corrections to the lightest Higgs mass. Due to these two effects the derived NMSSM upper bound on the lightest Higgs mass is about (20-25)GeV lower than the one, reported in literature previously.
There is a large difference between the MSSM and the NMSSM constraints on the lightest Higgs mass. Namely, the upper bound on the lightest Higgs mass in the NMSSM can be about 40GeV larger than in the MSSM. Such a large difference occurs for $\tan\beta$ being close to 2. Due to this difference the NMSSM with the low $\tan\beta$ scenario is, generally speaking, far out from being excluded. Notice however that the NMSSM upper bound on the lightest Higgs mass is saturated in the narrow region of parameters space, where the singlet Higgs sector is decoupled and the detectable Higgs bosons masses are described by the MSSM-type mass matrices. To compare the models predictions in wider region of parameters space, the predictions for the supersymmetry breaking parameters of theories

are necessary.

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APPENDIX

At tree level the 3×3 symmetric mass matrix for the fields Φ_1, Φ_2 and N_1 is the following:

$$\begin{aligned}
M^2_{S_{11}} &= M_Z^2\cos^2 2\beta+\lambda^2\eta^2\sin^2 2\beta\\
M^2_{S_{12}} &= -(M_Z^2-\lambda^2\eta^2)\sin 2\beta\cos 2\beta\\
M^2_{S_{13}} &= 2\lambda^2 v_3\eta+(A_\lambda+2\kappa v_3)\lambda\eta\sin 2\beta\\
M^2_{S_{22}} &= (M_Z^2-\lambda^2\eta^2)\sin^2 2\beta-2\lambda v_3\frac{A_\lambda+\kappa v_3}{\sin 2\beta}\\
M^2_{S_{23}} &= -(A_\lambda+2\kappa v_3)\lambda\eta\cos 2\beta\\
M^2_{S_{33}} &= 4\kappa^2 v_3^2+\kappa v_3 A_\kappa-\frac{A_\lambda\lambda\eta^2\sin 2\beta}{2v_3}
\end{aligned} \tag{A.1}$$

where A_λ, A_κ are trilinear SUSY breaking parameters of the Higgs potential [10, 11, 13, 18], v_3 is the singlet vev. The fields Φ_1, Φ_2, N_1 are given by equation (4).

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